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Genetic Algorithm for Chinese Postman Problems

Jiang Hua, Kang Li-shan†

State Key Laboratory of Software Engineering, Wuhan University, Wuhan 430072, Hubei, China

Abstract: Chinese Postman Problem is an unsettled graphic problem. It was approached seldom by evolutionary computation. Now we use genetic algorithm to solve Chinese Postman Problem in undirected graph and get good results. It could be extended to solve Chinese postman problem in directed graph. We make these efforts for exploring in optimizing the mixed Chinese postman problem.

Key words: Chinese postman problem; Eularian graph; genetic algorithm; evolutionary computation

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0 Introduction

A postman who wishes to travel along every road to deliver letters and come back to postoffice will find the shortest route. This problem is proposed by Kwan Mei-ko in 1962[1], was called the Chinese postman problem (CPP). This problem has many applications, including robot exploration, analyzing interactive system and web site usability. It was an unsettled problem and was approached seldom by evolutionary computation. Now we introduce how to use genetic algorithm to solve Chinese postman problem in undirected multi-graph and discuss how to modify it to solve the problem in directed multi-graph. Our aim is to explore how to solve the mixed Chinese postman problem, which is known to be NP-complete[2].

The origin of evolutionary computation was an attempt to mimic some of the processes taking place in natural evolution[3]. An Evolutionary Algorithm is an iterative and stochastic process that operates on a set of individuals (population) which represents a potential solution to the problem being solved. The iterative process will not come to stop until the optimal individual is present. Genetic algorithm is efficient in approaching the routing problem such as TSP.

1 Definitions

An undirected graph $G=(\boldsymbol{V},\boldsymbol{E})$ consists of a set of $\boldsymbol{V}$ of vertices and a multiset $\boldsymbol{E}$ of undirected edges. Each edge $e_i=(u,v)$ connects two vertices in $\boldsymbol{V}$ and has positive weight $w(e_i)$. The Chinese postman problem requires that all the edges in an undirected graph be included at least once in a tour of

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Biography: Jiang Hua(1974-), male, Master candidate, research direction: Evolutionary computation. E-mail: hua_j@whu.edu.cn

† To whom correspondence should be addressed

minimum costs. There exists a tour using each edge of the graph just once if the following property is satisfied : The degree of each vertex must be even. Thus the method of solution for Chinese postman problem consists of finding a minimum cost augmentation of the graph that satisfies the sufficient property and then identifying the tour over the augmented graph. The model is shown below:

Find : $E' \subset E$

Min: $\sum w(e'), e' \in E'$,

Subject to : $\forall v_i \in V, d(v_i) \in E_v$

Then $G' = (V, E \cup E')$ is the solution.

2 Genetic Algorithm for Chinese Postman Problem

Genetic algorithm is described as follows:

1) $t := 0$, generate an initial population $P(0)$;

2) Calculate the fitness of the individuals in $P(0)$;

3) while (NOT Termination_Criterion)

4) { using select function to create next parent generation $P(t)$ by individuals' fitness;

5) produce newly-born children generation $P'(t)$ by mutation and crossover;

6) choose next generation $P(t+1)$ from $P(t) \cup P'(t)$ by select function;

7) $t := t+1$;

}

Now we introduce the each part of the genetic algorithm for Chinese postman problem, such as genotype, fitness function and operators of mutation crossover and selection.

◆ Genotype:

Our task is finding the minimum cost augmentation of the graph. We know that the total number of the vertices which have odd degrees is even. Thus we may use the code method as follows:

1) Divide total m odd vertices into $m/2$ pairs and add the shortest route between each pair to eliminate all odd vertices.

2) Adjust the scheme of the partnership to minimize the total cost of the augmentation of the graph.

All odd vertices in the graph were picked out. Then we calculate the shortest route between each pair by Dijkstra's shortest path algorithm[4] and put the result in the matrix $\boldsymbol{M}$:

$$\boldsymbol{M} = \begin{bmatrix} a_{00}, e_{00} & \cdots & a_{00}, e_{0n} \\ \vdots & \cdots & \vdots \\ a_{n0}, e_{n0} & \cdots & a_{m}, e_{m} \end{bmatrix}$$

In $\boldsymbol{M}$, a_{ij} is the shortest distance between these vertices v_i and v_j and e_{ij} is the vector $\langle v_i, \cdots, v_j \rangle$ which is the shortest path between v_i and v_j . Now the question is how to couple these odd vertices to minimize $\sum w(e'), e' \in V'$.

The genotype is a vector $(v_1, v_2, \cdots, v_m)$ in which v_i is a odd vertex, v_{2i-1} and v_{2i} is a couple.

◆ Initialization:

In the step of initialization we assign one of m odd vertices to v_i randomly.

◆ Mutation:

Produce two random integer $i, j \in [1, n]$ and exchange the value of v_i and v_j to make a new partnership scheme.

◆ Crossover:

We choose individuals in parent generation by roulette wheel selection and choose pairs in individual randomly. Then exchange the pairs between individuals and adjust the rest pairs to validate the augment route. Thus the new population is produced.

◆ Fitness function:

We retrieve the shortest distance between vertices in each pair in matrix $\boldsymbol{M}$ and calculate the total cost of the augmentation of the graph as the fitness of the individuals.

◆ Select scheme:

We use roulette wheel selection which makes individuals equally selected to ensure the variety of the population.

3 Experiments

We applied the algorithm on a set of multigraphs and got the optimal solutions. For example, Fig. 1 is the original graph in which $a, d, f, g, k, m, n, o, w, x, y$ and z are odd vertices, Fig. 2 is the optimal solution in which $(a, x), (d, e, y), (f, g), (k, j, h, w)$, (m, n), (o, l, z) are the augment routes.

4 Use this Algorithm to Solve Chinese Postman Problem in Multidigraph

An multidigraph graph $G = (\boldsymbol{V}, \boldsymbol{A})$ is a set of $\boldsymbol{V}$ of vertices along with a multiset $\boldsymbol{A}$ that connect the vertices in $\boldsymbol{V}$. An arc (u, v) is an ordered pair of vertices in $\boldsymbol{V}$ that

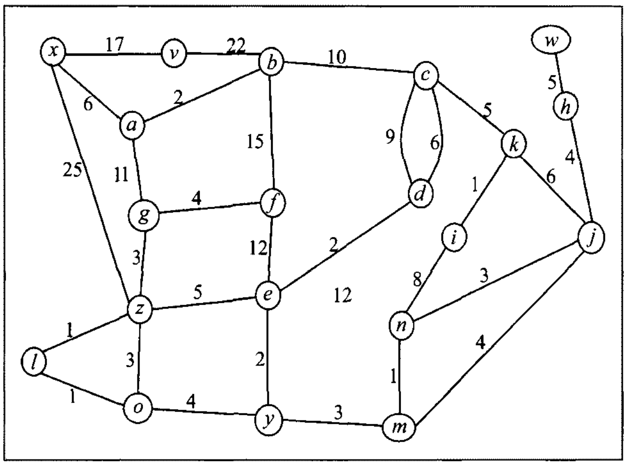

Fig. 1 The original graph

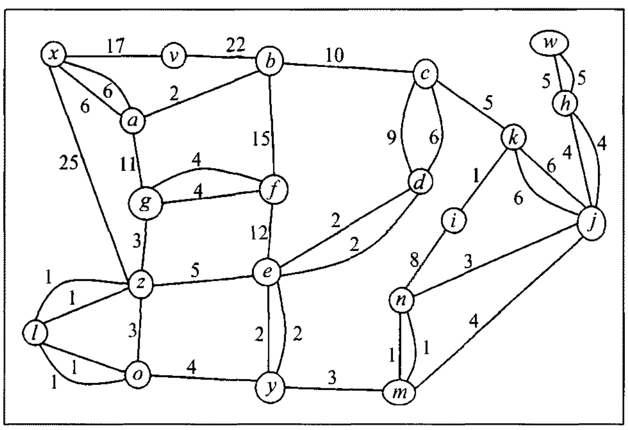

Fig. 2 The optimal solution

are connected in G and has positive weight $w(a_i)$. There exists a tour using each edge of the graph just once if the following property is satisfied: The indegree of each vertex must equal the outdegree. Thus the method of solution for Chinese postman problem consists of finding a minimum cost augmentation of the graph that satisfies the sufficient property and then identifying the tour over the augmented graph. The model is shown as follow:

Find : $A' \subset A$

Min: $\sum w(a'), a' \in A'$,

Subject to : $\forall v_i \in V, d(v_i) \in E_v$

Then $G' = (V, A \cup A')$ is the solution.

We usually convert Chinese postman problem in multidigraph to a minimum cost maximum flow problem. In this way the program will perform badly when the network becomes bigger and more complex. But our genetic algorithm still keeps efficient. What we should do is only change the definition of gene code, not need to change the scheme of the genotype.

In multidigraph the sum of the indegrees of the vertices equals sum of the outdegrees. Now let's modify the definition of the genotype, the vector $(v_1, v_2, \cdots, v_m)$: a vertex of which the indegrees exceeds its outdegrees was assigned to v_{2i-1}, a vertex of which the outdegrees exceeds its indegrees was assigned to v_{2i}. The number of the assignments of an unbalanced vertex in the vector is $|d(v_i)_{\text{in}} - d(v_i)_{\text{out}}|$. Now we can optimize the Chinese postman problem without modifying the framework of our genetic algorithm.

5 Conclusion

This paper proposes a scalable genetic algorithm for Chinese postman problem in undirected and directed multigraph. It keeps efficient in great and complex network. In further research we may modify it to get optimal solution of mixed Chinese postman problem for which there is still not an efficient algorithm.

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